Bachelor of Science (B.Sc.) Semester-I (C.B.S.) Examination

MATHEMATICS (M_1 : Algebra and Trigonometry)

Compulsory Paper—I

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Solve all **FIVE** questions.

(2) All questions carry equal marks.

(3) Question No. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$ by reducing into the normal form. 6

(B) Investigate the values of λ and μ so that the equations $x + 2y + z = 8$, $2x + y + 3z = 13$, $3x + 4y - \lambda z = \mu$ have (i) no solution, (ii) a unique solution and (iii) infinite number of solutions. 6

OR

(C) Find the eigen values and corresponding eigen vectors of the matrix :

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$ 6

(D) Verify Cayley-Hamilton theorem for the matrix :

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}.$$ 6

UNIT—II

2. (A) Solve the equation $x^3 - 13x^2 + 15x + 189 = 0$ having given that one root exceeds the other by 2. 6

(B) Solve the reciprocal equation :

$$x^4 - 10x^3 + 25x^2 - 10x + 1 = 0.$$ 6

OR

(C) Solve the cubic equation $x^3 - 21x - 344 = 0$ by Cardon's method. 6

(D) Solve the biquadratic equation :

$x^4 - 3x^3 + x^2 - 2 = 0$ by Ferrari's method. 6

UNIT—III

3. (A) State De Moivre's theorem and by using it find all the values of $(32)^{1/5}$. 6

(B) If $x_r = \cos\left(\frac{\pi}{2^r}\right) + i\sin\left(\frac{\pi}{2^r}\right)$, then prove that :

$$x_1 x_2 x_3 \ldots \text{ ad inf.} = -1.$$ 6

OR

(C) Separate the real and imaginary parts of $\tan^{-1}(x + iy)$. 6

(D) Prove that : 6

(i) $\cosh^{-1} x = \log[x + \sqrt{x^2 - 1}]$

(ii) $\tanh^{-1} x = \sinh^{-1} \frac{x}{\sqrt{1 - x^2}}$.

UNIT—IV

4. (A) Define : Group (G, 0), order of a group (G, 0). Show that a set $G = \{1, w, w^2\}$, where $w^3 = 1$ forms an abelian group of order 3 under multiplication. 6

(B) Prove that intersection of any two subgroups of a group G is a subgroup of G. Also prove by giving an example that the union of two subgroups is not necessarily a subgroup. 6

OR

(C) State and prove Lagrange's, theorem for a group. 6

(D) Prove that every permutation can be expressed as a product of transpositions. Express the permutation

$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 1 & 6 & 5 & 7 & 8 \end{pmatrix}$ as the product of transpositions. 6

UNIT—V

5. (A) Reduce the matrix $A = \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 6 \\ 1 & 5 & 0 \end{bmatrix}$ to normal form. 1½

(B) If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, then find the characteristic equation for A. 1½

(C) Show that the equation $x^4 - 6x^3 + 8x^2 - 30x + 25 = 0$ has no any imaginary root. 1½

(D) Find the condition that the sum of two roots of the equation $x^3 - px^2 + qx - r = 0$ is zero. 1½

(E) Prove that :

(i) $\log(-1) = \pi i$ and

(ii) $\log(i) = \frac{\pi}{2} i$. 1½

(F) Prove that :

$$e^{i\theta} = \cos\theta + i\sin\theta \text{ and}$$
$$e^{-i\theta} = \cos\theta - i\sin\theta$$ 1½

(G) Prove that the identity element of a group G is unique. 1½

(H) Write the permutation $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}$ as the product of disjoint cycles. 1½